

On balance of information in bipartite quantum communication systems: entanglement-energy analogy

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We adopt the view according to which information is the primary physical entity that possesses objective meaning. Basing on two postulates that (i) entanglement is a form of quantum information corresponding to internal energy (ii) sending qubits corresponds to work, we show that in the closed bipartite quantum communication systems the information is conserved. We also discuss entanglement-energy analogy in context of the Gibbs-Hemholtz-like equation connecting the entanglement of formation, distillable entanglement and bound entanglement. Then we show that in the deterministic protocols of distillation the information is conserved. We also discuss the objectivity of quantum information in context of information interpretation of quantum states and algorithmic complexity.

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I. INTRODUCTION

It is astonishing that just lately after over sixteen years quantum formalism reveals us new possibilities due to entanglement processing being a root of new quantum phenomena such as quantum cryptography with Bell theorem [1], quantum dense coding [2], quantum teleportation [3], quantum computation [4]. It shows how important is to recognize not only the structure of the formalism itself but also potential possibilities encoded in it.

In spite of many beautiful experimental and theoretical results on entanglement there are still difficulties in understanding its many faces. It seems to be a reflection of basic difficulties in the interpretation of quantum formalism as well as quantum-classical hybridism in our perception of Nature. To overcome the latter the existence of unitary information field being a necessary condition of *any* communication (or correlation) has been postulated [5–7] as well as the information interpretation of quantum wavefunction has been considered [6]. It rests on the generic information paradigm according to which the notion of information represents a basic category and it can be defined independently of probability itself [8,11,12,7]. It implies that Nature is unbroken entity. However, according to double, hylemorphic nature of the unitary information field, there are two mutually coupled levels of physical reality in Nature: logical (informational) due to potential field of alternatives and energetic due the field of activities (events) [13]. Then from the point of view of the generic information paradigm, quantum formalism is

simply a set of extremely useful informational algorithms describing the above complementary aspects of the same, really existing unitary information field. It leads in a natural way to analogy between information (entanglement) and energy being nothing but a reflection of unity of Nature.

Following this route, one attempts to find some useful analogies in the quantum communication domain. Namely, physicists believe that there should exist the laws governing entanglement processing in quantum communication systems, that are analogous to those in thermodynamics.

Short history of this view has its origin in the papers by Bennett *et al.* who announced a possible irreversibility of the entanglement distillation process [14,15]. Popescu and Rohrlich [16] have pointed out analogy between distillation-formation of pure entangled states and Carnot cycle, and they have shown that entanglement is extensive quantity. The authors formulated principle of entanglement processing analogous to the second principle of thermodynamics: “*Entanglement cannot increase under local quantum operations and classical communication*”. Vedral and Plenio [17] have considered the principle in detail and pointed out that there is some (although not complete) analogy between efficiency of distillation and efficiency of Carnot cycle. In Refs [18,21] entanglement-energy analogy has been developed and conservation of information in closed quantum systems has been postulated in analogy with the first principle of thermodynamics: *Entanglement of compound system does not change under unitary processes on one of the subsystems* [18]. Then an attempt to formulate the counterpart of the second principle in a way consistent with the above principle has been done (since in the original Popescu-Rohrlich formulation entanglement was not conserved).

The main purpose of the paper is to support entanglement-energy analogy by demonstration that in the closed bipartite quantum communication system the information is conserved. The paper is organised as follows. In section II we describe closed quantum communication bipartite system. The next section contains formal description of balance of quantum information involving notions of physical and logical work. In section IV we introduce the concept of useful logical work in quantum communication. In next section we present balance of information in teleportation. In section VI we discuss entanglement-analogy in the context of the Gibbs-Hemholtz-like equation connecting entanglement

of formation, distillable entanglement and bound entanglement. In section VII we present the balance of information in the process of distillation. In the last section we discuss the objectivity of quantum information in context of information interpretation of quantum states and algorithmic complexity.

II. CLOSED QUANTUM COMMUNICATION SYSTEM: THE MODEL

Consider closed quantum communication (QC) system U composite of system S , measuring system M and environment R

$$U = S + M + R \quad (1)$$

where each system is split into Alice and Bob parts S_X, M_X, R_X ; $X = A, B$.

It is assumed that Alice and Bob can control the system S_X which does not interact with environment R_X . The M_X system consists of m_X qubits and continuously interacts with environment R_X . In result the system M_X , playing the role of “ancilla”, is measured in distinguished basis $|x_1 x_2 \dots\rangle$, $x_i = 0, 1$ [23]. In this sense the measurement is understood here as *the process of irreversible entanglement with some environment* and the system R_X is to ensure this irreversibility. Note that in the above approach the evolution of the system is unitary i. e. abandon the von Neumann projection postulate which leads to violation of energy-momentum conservation [19]. Then acting on one part of entangled system, we have no way to *annihilate* entanglement. The latter can change only by means of interacting of the *both* entangled subsystems. It may be objected that we can destroy entanglement e.g. by randomizing the relative phases on the subsystems of interest. However, if the reduction of wave packet is *not* regarded to be a real physical process, then the above operation must be considered as entangling the subsystem with some other system by means of a *unitary* transformation. Then the entanglement will not vanish but it will *spread* over all the three subsystems.

The operations Alice and Bob can perform in our QC system are:

- Quantum communication: Alice and Bob can exchange particles from the system S_X .
- “Classical communication”: Alice and Bob can exchange particles from the system M_X

Note that the number of qubits of the systems S_A and S_B can change but the total number of qubits of the system M is conserved (similarly for S). Besides Alice and Bob can perform unitary transformation over the system $M_X + S_X$, $X = A, B$.

We would like to stress one more that in our approach the measurement represents an irreversible entanglement

rather than the “projection” of the state. To see it consider the case when Alice and Bob share a singlet state and Alice performs a measurement on it. The initial state of the system $M_A + S_A + S_B$ (M_A represents the Alice’s ancilla while S_A, S_B correspond to the particles forming a singlet state) is

$$|\Psi\rangle_{M_A S_A S_B} = |0\rangle_{M_A} |\Psi_A^{singlet}\rangle = |0\rangle_{M_A} \frac{1}{\sqrt{2}} (|0\rangle_{S_A} |1\rangle_{S_B} - |1\rangle_{S_A} |0\rangle_{S_B}) \quad (2)$$

Then Alice performs the unitary operation U on subsystem $M_A + S_A$. This operation corresponds to the interaction between M_A and S_A and can be represented by C-NOT gate. As a result the whole system is in the state

$$|\Psi'\rangle_{M_A S_A S_B} = \frac{1}{\sqrt{2}} (|0\rangle_{M_A} |0\rangle_{S_A} |1\rangle_{S_B} - |1\rangle_{M_A} |1\rangle_{S_A} |0\rangle_{S_B}) \quad (3)$$

Further M_A can be irreversibly entangled with environment system R_A (which models the irreversibility of the measurement). But R_A is still on Alice side, hence we have entanglement between systems ($R_A + M_A + S_A$) and S_B unchanged and equal to $E = 1$ e-bit.

Of course, there are some interpretational problems if one imagines that Alice “reads out” the result of the measurement as then we encounter problems coming from possible extension of the model by the projection postulate. However that for practical reasons (i. e. as far as quantum information qualitative description is concerned) the informational processes like e. g. quantum teleportation do not require reading the data. Moreover, it must be noted that at the absence of the projection postulate the above model can be viewed as consistent with “many worlds” interpretation [20].

III. CONSERVATION OF QUANTUM INFORMATION: FORMAL DESCRIPTION

To determine balance of information in the closed system U we adopt two basic postulates [18,21]

1. Entanglement is a form of quantum information corresponding to internal energy.
2. Sending qubits corresponds to work.

In accordance with the postulate 1, the information is physical quantity that, in particular, should be *conserved* in closed quantum systems, similarly as energy. The second postulate allows to deal with communication *processes* (in thermodynamics work is a functional of process). To obtain the balance we must define our “energy” and “work” quantitatively. To this end consider system X described in the Hilbert space \mathcal{H} , $\dim \mathcal{H} = d$ being in a state ϱ_X . We define *informational content* I_X of the state ϱ_X as follows (cf. [24]):

$$I_X = \log \dim \mathcal{H} - S(\varrho_X) \quad (4)$$

where $\dim \mathcal{H} = d$, $S(\varrho_X) \equiv S(X)$ are the dimension of the Hilbert space and the von Neumann entropy of the system state. Note that I_X satisfies the inequality $0 = I_X^{min} \leq I_X \leq I_X \leq I_X^{max} = \log \dim \mathcal{H}$ where I_X^{min} and I_X^{max} are the information content of the maximal mixed state and pure state respectively. Thus it is well defined quantity which measures informational content of the system ϱ_X .

The formula (4) needs some comment as usually one interprets the von Neumann entropy as a measure of information. In fact there is no contradiction. Imagine for a moment that we admitt projection postulate i. e. Alice knows the concrete result of the measurement. Then the von Neumann entropy measures the information gain *after* the measurement while the formula (4) corresponds to the information *prior* the measurement and this information, in particular, is maximal if the system is in pure state. This is the reason while we use the name informational *content* as it has actual rather than potential (i. e. related to the future measurement) character. Below we shall see that, after we abandon the projection postulate, the above formula allows to perform a balance of quantum information in a consistent way. Note that the Hilbert space dimension used in formula (4) is present also in definitions of other notions (see below), in particular in the case of useful logical work (sec. IV). It plays, to some extent, the role similar to the one in channels capacities theory or error correction codes where dimension of “error free” subspace is a central notion.

Consider now the QC system U , being in the initial pure state ψ_{in} , described by general Alice-Bob Hilbert space scheme as follows

$$\left. \begin{array}{c} \mathcal{H}_A \\ \otimes \\ \mathcal{H}_{A'} \end{array} \otimes \mathcal{H}_B \right\} \psi_{in}, \quad (5)$$

where $\mathcal{H}_A \otimes \mathcal{H}_{A'}$, \mathcal{H}_B are the Hilbert spaces of the $S_A + M_A + R_A$ and $S_B + M_B + R_A$ respectively. Then in accordance with (4) the information contents of the Alice and Bob subsystems are defined as follows

$$I_A = \log \dim(\mathcal{H}_A \otimes \mathcal{H}_{A'}) - S(A + A'); \quad (6)$$

$$I_B = \log \dim \mathcal{H}_B - S(B), \quad (7)$$

where $\dim(\mathcal{H}_A \otimes \mathcal{H}_{A'})$ and $\dim \mathcal{H}_B$ are the dimensions the corresponding Hilbert spaces while $S(A + A')$, $S(B)$ are the von Neumann entropies of the subsystems.

Now, after transmission of the system A' to receiver (Bob) the Alice-Bob Hilbert space scheme is given by

$$\left. \begin{array}{c} \mathcal{H}_A \\ \otimes \\ \mathcal{H}_{A'} \end{array} \otimes \mathcal{H}_B \right\} \psi_{out} \quad (8)$$

and the total sytem U is in the final state ψ_{out} .

Now, in accordance with the above “sending qubits – work” postulate we consider *physical work* performed over the system U being a *physical transmission* of particles. Consequently, we define W_p as a number of sent qubits of the system A'

$$W_p = \log \dim \mathcal{H}_{A'}. \quad (9)$$

Note that after transmission of the system A' to the Bob, there is increase of the information content of his subsystem. Then we say that the system U performed the *logical work* W_l that is defined as increase of the informational content of the Bob (in general - receiver) system.

$$W_l = I_{out}^B - I_{in}^B \quad (10)$$

where $I_{in}^B = I^B$, $I_{out}^B = I^{B+A'}$. Then one can regard the physical work as sending “matter” while the logical work – sending “form” that is consistent with the assumed hylemorphic nature of the information field. Subsequently we can define initial and final entanglement of the system U as

$$E_{in} = S(B) = S(A + A'); \quad E_{out} = S(A) = S(B + A'), \quad (11)$$

where obvious relations between the entropies of the subsystems hold. Now, in accordance with the first postalate, E_{in} and E_{out} are simply initial and final *potential* informations contained in the total system. Having so defined quantities it is not hard to obtain the following information balance equations

$$E_{in} + W_p = E_{out} + W_l \quad (12)$$

or equivalently

$$I_{in}^A + I_{in}^B + 2E_{in} = I_{out}^A + I_{out}^B + 2E_{out} = const. \quad (13)$$

Note that the latter equation is compatible with the principle of information conservation expressed in the following form (equivalent to the one in the Introduction): *For a compound quantum system a sum of information contained in the subsystems and information contained in entanglement is conserved in unitary processes* [18].

To see how the above formalism works, consider two simple examples with ideal quantum transmission. Suppose, Alice sends an *unentangled* qubit of the system S to Bob. Then the physical work W_p is equal to 1 qubit. In result the informational content of Bob’s system increases by 1, thus also the logical work W_l amounts to one qubit. Of course, in this case both “in” and “out” entanglement are 0.

Suppose now that Alice sends maximally *entangled* qubit to Bob. Here, again, physical work is 1 qubit, and there is no initial entanglement. However the final entanglement is one ebit and logical work is 0, because the state of the Bob system is now completely mixed.

Now we see that, according to the balance equation (12) the difference $W_p - W_l$ between the physical and

logical work is due to entanglement. Indeed, as in the above example, sending particle may result in increase of entanglement rather than performing nonzero logical work.

IV. USEFUL LOGICAL WORK: QUANTUM COMMUNICATION

The basic question arises in the context of quantum communication. Does the balance (12) distinguish between quantum and “classical” communication in our model? It follows from definition that the physical work does not distinguish between these types of communication. But what about logical work? Suppose that Alice sent to Bob a particle of the system M_A in a pure state $|0\rangle$. But in our model such state does not undergo decoherence. Then the logical work W_l is equal to one qubit. Needless to say it is not quantum communication. Hence the logical work is not “useful” in this case.

In quantum communication we are usually interested in sending faithfully any superpositions without decoherence. Therefore it is convenient to introduce the notion of *useful logical work* as follows.

Definition. Useful work is amount of qubits of the system S transmitted without decoherence

$$W_u = \log \dim \mathcal{H}, \quad (14)$$

where \mathcal{H} is the Hilbert space transmitted asymptotically faithfully. The latter means that *any* state of this space would be transmitted with asymptotically perfect fidelity. We see that the work performed in previous example was not useful, since in result of the process, only the states $|0\rangle$ or $|1\rangle$ can be transmitted faithfully.

V. BALANCE OF INFORMATION IN TELEPORTATION

To see how the above formalism works, consider the balance of quantum information in teleportation [3] [25]. Now the system S_A consists of a particle in unknown state and one particle from maximally entangled pair, whereas the second particle from the pair represents S_B system. The system M_A consists of two qubits that interact with environment R_A (Fig. 1).

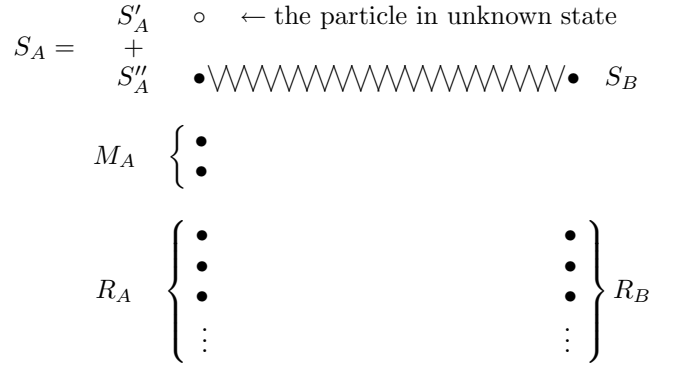


FIG. 1. The model of quantum communication system

The latter is only to ensure effective irreversibility of the measurement and it is demonstrable that its action is irrelevant to the information balance in the case of teleportation. As one knows, the initial state can be written in the following form

$$\psi_{in} \equiv \psi_0 = \psi_{S'_A}^{unknown} \otimes \psi_{S''_A S_B}^{singlet} \otimes |00\rangle_{M_A}, \quad (15)$$

where $\psi_{S'_A}^{unknown}$ is the state to be teleported, $\psi_{S''_A S_B}^{singlet}$ is the singlet state of entangled pair and $|00\rangle_{M_A}$ is the initial state of the measuring system. It is easy to check that the initial entanglement E_{in} of the initial state is equal to one e -bit. Now Alice performs “measurement” being local unitary transformation on her joint system $S_{A'} + S_{A''} + M_A$. In result ψ_{in} transforms to

$$\psi_1 = \frac{1}{2} \sum_{i=0}^3 \psi_{S'_A S''_A}^i \otimes \psi_B^{i(unknown)} \otimes |i\rangle_{M_A}, \quad (16)$$

where $\psi_{S'_A S''_A}^i$ constitute Bell basis, $\psi_B^{i(unknown)}$ is rotated $\psi_{S'_A S''_A}^{unknown}$, $|i\rangle_{M_A}$ is the state of the system M_A indicating the result of the measurement (i -th Bell state obtained). Since the Alice’s operation is unitary one, it does not change initial asymptotic entanglement. Subsequently, Alice sends the two particles of the system M_A to Bob. In accordance with definition (6), it corresponds to two qubits $W_p = 2$ of work performed over the system. At the same time the state ψ_1 transforms to ψ_2 of the form

$$\psi_2 = \frac{1}{2} \sum_{i=0}^3 \psi_{S'_A S''_A}^i \otimes \psi_B^{i(unknown)} \otimes |i\rangle_{M_B}. \quad (17)$$

Finally Bob decouples the system S_B from other ones by unitary transformation that of course does not change the asymptotic entanglement.

After classical communication from Alice entanglement of the total system increased to the value $E_{out} = 2$ e-bits. Indeed Alice sends two particles of system M_A to Bob which are entangled with particles S'_A, S''_A . On the other hand, the logical work performed by the system in the above process amounts to $W_u = 1$. One can see the

balance equation (12) is satisfied, and is of the following form

$$(E_{in} = 1) + (W_p = 2) = (E_{out} = 2) + (W_u = 1) \quad (18)$$

One easily recognizes the result of the logical work in the transmission of the unknown state to Bob. Since it is faithfully transmitted independently of its particular form, we obtain that also useful logical work W_u is equal to 1 qubit. Hence in the process of teleportation all the work performed by the system is useful, and represents quantum communication.

VI. THERMODYNAMICAL ENTANGLEMENT-ENERGY ANALOGY. GIBBS-HEMHOLTZ-LIKE EQUATION

So far we have considered balance of information in closed QC system. For open system (being, in general, in mixed state) the situation is much more complicated being a reflection of fundamental irreversibility in the asymptotic mixed state entanglement processing [14,15,26,27]. Namely it has been shown [26] there is a discontinuity in the structure of noisy entanglement. It appeared that there are at least two quantitatively different types of entanglement: free - useful for quantum communication, and bound - nondistillable, very weak and peculiar type of entanglement. In accordance with entanglement-energy analogy this new type of entanglement was defined by equality

$$E_F = E_{bound} + E_D, \quad (19)$$

where E_F and E_D are asymptotic entanglement of formation [21,22] and distillable entanglement [29] respectively. Note that for pure entangled states $|\Psi\rangle\langle\Psi|$ we have always $E_F = E_D$, $E_{bound} = 0$ [15]. Then, in this case the whole entanglement can be converted into the useful quantum work (see Fig. 2a) with $E \equiv E_F(|\Psi\rangle\langle\Psi|)$. For bound entangled mixed states we have $E_D = 0$, $E_F = E_{bound}$. It is quite likely that $E_F > 0$ (so far we know only that $E_f > 0$ [28]). Here E_f is entanglement of formation defined in Ref. [29]. and then all prior non-trivial entanglement of formation would be completely lost. Thus in any process involving only separable or bound entangled states useful quantum work is just zero. In general, however, it can happen that the state contains two *different* types of entanglement. Namely there are cases where E_{bound} is strictly positive i. e. we have [27,30]

$$E_{bound} = E_F - E_D > 0. \quad (20)$$

This reveals fundamental irreversibility in the domain of quantum asymptotic information processing [31]. It can be viewed as an analogue to irreversible thermodynamical processes where only the free energy (which is not equal to the total energy) can be converted to

useful work. This supports the view [21] according to which the equation (19) can be regarded as quantum information counterpart of the thermodynamical Gibbs-Helmholtz equation $U = F + TS$ where quantities E_F , E_D , E_{bound} correspond to internal energy U , free energy F and bound energy TS respectively (T and S are the temperature and the entropy of the system).

The above entanglement-energy analogy has lead to the extension [32] of the “classical” paradigm of LOCC operations by considering new class of entanglement processing called here entanglement enhanced LOCC operations (EELOCC). In particular, it suggested that entanglement can be pumped from one to other system producing different nonclassical chemical-like type processes. In fact it allowed to find a new quantum effect - *activation* of bound entanglement that corresponds to chemical activation process [32]. Similarly, a recently discovered *catalysis* of pure entanglement involves EELOCC operations [33]. In result the second principle of entanglement processing (see Introduction) has been generalized [34] to cover the EELOCC paradigm: *By local action, classical communication and N qubits of quantum communication, entanglement cannot increase more than N e-bits.*

Now, it is interesting in the above context to consider the problem of information balance in the cases where systems are in mixed states.

VII. BALANCE OF INFORMATION IN DISTILLATION PROCESS

So far in our balance analysis the initial state of the QC system was pure. Let us consider the more general case. Suppose that initial state of the system S is mixed. We have not generalized formalism to such case. We can however perform balance of information in the case of the distillation process [14] (see in this context [35]). This task would be, in general, very difficult, because the almost all known distillation protocols are *stochastic*. As one knows, the distillation protocol aims at obtaining singlet pairs from a large amount of noisy pairs (in mixed state) by LOCC operations. A convenient form of such a process would be the following: Alice and Bob start with n pairs, and after distillation protocol, end up with m singlet pairs. Such a protocol we shall call *deterministic*. Unfortunately, in the stochastic protocols the situation is more complicated: Alice and Bob get with some probabilities different number of output distilled pairs:

$$\varrho_{in} = \underbrace{\varrho \otimes \varrho \otimes \cdots \otimes \varrho}_n \rightarrow \begin{cases} \rightarrow p_0, \text{ no output singlets} \\ \rightarrow p_1, \text{ one output singlet} \\ \rightarrow p_2, \text{ two output singlet} \\ \vdots \end{cases}$$

Since we must to describe the process in terms of closed system, we will not see the above probabilities, but only

their amplitudes. As a result, we will have *no* clear distinction between the part of the system containing distilled singlet pairs and the part containing the remaining states of no useful entanglement.

Consider for example the first stage of the Bennett *et al.* [3] recursive protocol. It involves the following steps

- take two two-spin 1/2 pairs, each in input state ϱ
- perform operation $XOR \otimes XOR$
- measure locally the spins of the target pair, and:
 - if the spins agree (probability p_a), keep the source pair
 - if the spins disagree (probability p_d), discard both pairs

After this operation we have the following final “ensemble”

$$\{(p_a, \text{one pair in a new state } \tilde{\varrho}), (p_d, \text{no pairs})\}$$

If we include environment to the description, the events “no pair” and “one pair in state $\tilde{\varrho}$ ” will be entangled with states of measuring apparatuses (and environment) indicating these events. Then we see, that our total system becomes more and more entangled in a various possible ways, so that it is rather impossible to perform the balance of information.

Fortunately, in a recent work Rains [35] showed that any distillation protocol can be replaced with a deterministic one, achieving the same distillation rate:

$$\varrho^{\otimes n} \rightarrow \varrho_{out} \simeq |\psi_{distilled}\rangle\langle\psi_{distilled}| \otimes \varrho_{rejected}$$

where $\psi_{distilled}$ is the state of m distilled singlet pairs while $\varrho_{rejected}$ is the state of the rejected pairs. In this case the system can be divided into two parts

$$S = S_{distilled} + S_{rejected} \quad (21)$$

where $S_{distilled}$ is disentangled with the rest of universe $S_{rejected}$ is entangled with M , hence also with environment R .

This possibility of the clear partition into two systems is crucial for our purposes. Now the whole balance can be preformed in this case as follows. As an input we have the state ϱ with value of asymptotic entanglement of formation $E = E_F(\varrho)$. Because it is mixed we can take its purification adding come ancilla which would have the asymptotic entanglement $E' = E + (E' - E)$. Now we can perform the distillation process, having no access to the ancilla. After the process the state of our whole system is still separated according to the formula (21) but now the state $S_{rejected}$ involves the degrees of freedom of the ancilla. The balance of the information can now be easily performed taking, in particular, into account that distillable entanglement E_D can be interpreted as a useful work (14) W_u (Alice can always teleport through state $|\Psi_{distilled}\rangle\langle\Psi_{distilled}|$ if she wishes).

To make the balance fully consistent one should subtract from both input and output data the additional entanglement $E' - E$ coming from extension of the system to the pure state. As the input physical work (connected with optimal distillation protocol) *is the same* regardless of the value $E' - E$ and the kind of the ancilla itself, the whole balance is completely consistent. The input quantities of E , $\Delta = (E' - E)$ plus W_p as well as the output ones $E_D = W_u$, Δ , $E_{out} = E(\varrho_{rejected}) = E_{bound}$ are depicted on figure Fig. 2b. In particular if we deal with BE states then the corresponding diagram takes the form of Fig. 2c.

VIII. OBJECTIVITY OF QUANTUM INFORMATION: INFORMATION INTERPRETATION OF QUANTUM STATES

As we have dealt with balance of information in quantum composite systems it is natural to ask about objectivity of the entity we qualify. In this section we discuss that question and related ones in the context of quantum information theory and interpretational problems of quantum mechanics. As one knows the latter defends oneself very well against commonly accepted interpretation. In result a number of different interpretations permanently grows while there is no operational criterions (except, may be, Ockham reazor) to eliminate at least some of them.

It is characteristic that despite of dynamical development of interdisciplinary domain - quantum information there is no, to our knowledge, impact of the latter on interpretational problems. In this context a basic question arises: Does quantum information phenomena provide objective promises for existence of “natural” ontology inherent in quantum formalism?

It is interesting that from among discovered recently quantum effects just quantum cryptography provides answer “yes”. To see it clearly, consider quantum cryptographic protocol. A crucial observation is that the possibility of secret sharing key is due to the fact that we send quantum states *themselves* not merely the *classical information* about them! [36] Clearly, the latter could be cloned by the eavesdropper and it is reason for which all classical cryptographic schemes are, in principle, not secure. Then the use of qubits is *crucial* if we would like to take any advantage of the novel possibilities offered quantum information theory.

Now, as there are experimental implementations of quantum information protocols [37], it follows that quantum information is objective and it can provide natural ontological basis for interpretation of quantum mechanics. Then we arrive at important conclusion: *Quantum states carry two complementary kinds of information: the “classical” information involving quantum measurements and “quantum” information that can not be cloned* [?].

Note that it is consistent with proposed earlier information interpretation of the wave function in terms of

objective information content [6]. On the other hand it contradicts the Copenhagen interpretation according to which the wavefunctions have no objective meaning and only reality is the result of a measurement. It is remarkable that the above information interpretation of quantum states is compatible with the above mentioned unitary information field concept which rests in the assumption that information is physical [7,38,39] and can be defined independently of probability itself. First axiomatic definition of classical information “without probabilities” was considered by Ingarden and Urbanik [11]. Quantum version of the definition was introduced by Ingarden and Kossakowski [12]. On the other hand Kolmogorow [8], Solomonoff [9], Chaitin [10] introduced the concept of classical algorithmic information or complexity. Recently the classical algorithmic information was incorporated to the definition of the so called physical entropy being a constant of “motion” under the “demonic evolution” [40,41].

Quite recently algorithmic information theory was extended in different ways to quantum states by Vitanyi [42] and Berthiaume *et al.* [43]. In fact one can convince oneself that the approaches [42] and [43] correspond to the above complementary kinds of information associated with quantum state. Indeed, Vitanyi algorithmic complexity measures amount of “classical” information in bits necessary to approximate the quantum state. Needless to say, from the point of view of quantum cryptography such information is useless. On the other hand the bounded fidelity version of quantum Kolmogorow complexity measures amount quantum information in a qubit string and it is closely related to quantum compression theory [44–46].

IX. SUMMARY

In conclusion we have developed the entanglement – energy analogy based on some natural postulates: (i) entanglement is a form of quantum information being counterpart of internal energy, (ii) the process of sending qubits as a counterpart of work. We also assume that the evolution of the quantum system is unitary.

Basing on the above postulates we have considered the balance of quantum information for bipartite quantum communication systems i. e. the systems composed of two spatially separated laboratories endowed with classical informational channel plus local quantum operations. We have introduced the notion of informational content of quantum state being a difference of maximal possible von Neumann entropy and the actual one. Then we have defined physical work as a number of qubits physically sent from Alice to Bob. We have also defined logical work as an increase of the informational content of Bob state. To have a proper description of quantum communication processes we have also introduced a notion of useful logical work as amount of qubits transmitted

without decoherence.

Those tools have allowed us to perform the detailed balances of quantum information in two important processes of quantum communication: quantum teleportation and distillation of quantum noisy entanglement. In particular we have discussed the question of balance of quantum information for open systems. In the context of balance scheme and related notions we conclude that the irreversibility connected with existence of bound entanglement can be viewed as an analogue to irreversible thermodynamical processes where only the free energy (which is not equal to the total energy) can be converted to useful work. This allows us to interpret the equation for entanglement of formation as quantum information counterpart of the thermodynamical Gibbs-Helmholtz equation.

Finally we discuss the objectivity of quantum information in general context of some recent achievements of quantum information theory including quantum cryptography and recent propositions of classical and quantum algorithmic information. This leads us to the conclusion that quantum states reflect properties of quantum information as objective entity involving “classical” and “quantum” components which correspond to recently introduced “classical” and “quantum” algorithmic complexities. So the balance performed in the present paper concerns objective quantities rather than purely formal objects. We hope that the present informational approach to bipartite quantum communication systems, when suitably developed, may lead to deeper understanding of quantum information processing domain.

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FIG. 2. The diagram illustrating balance of quantum information in entanglement distillation process for: (a) pure states case, (b) general case, (c) bound entangled states case.

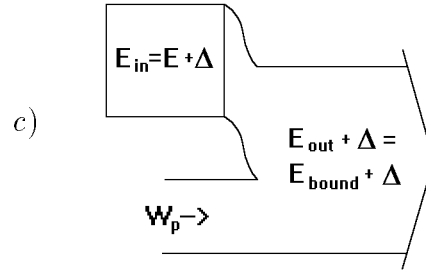
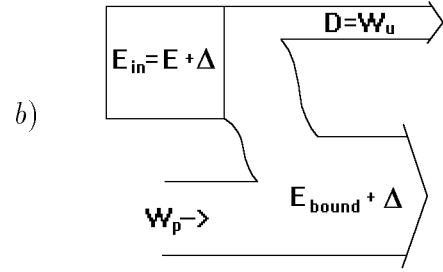
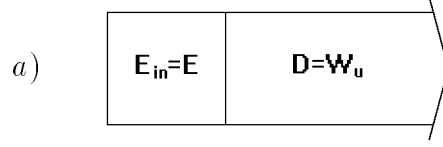


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